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Effects of uniform suction and surface tension on laminar filmwise condensation on a horizontal elliptical tube in a porous medium

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A B S T R A C T

This study performs a theoretical investigation into the problem of steady filmwise condensation flow over the external surface of a horizontal elliptical tube embedded in a porous medium with suction at the tube surface. The combined effects of the surface tension force and the gravitational force in driving the flow of the liquid film within the porous medium are modeled using Darcy's law. An effective suction function, *f*, is introduced to model the effect of the suction force at the wall on the thickness of the condensate film. The theoretical results presented in this study show that the heat transfer performance can be enhanced by applying a suction effect at the wall. Furthermore, it is shown that the surface tension force has a negligible effect on the mean Nusselt number.

1. Introduction

Vapor condensation on a horizontal tube in a porous medium has received extensive attention in the literature over the years due to its wide range of practical applications, ranging from heat exchange systems to thermally-enhanced oil recovery processes, waste disposal, chemical engineering processes, heat pipe design, geothermal energy utilization, and so forth.

The problem of film condensation on a flat vertical surface exposed to the ambient environment was originally analyzed by Nusselt [1] in 1916. In formulating the condensation problem, Nusselt imposed three basic assumptions, namely the condensate film was very thin, the convective and inertial effects were negligible, and the temperature had a linear profile within the liquid film. Later researchers developed more accurate analytical results for the condensation problem by removing these overly-restrictive assumptions, and extended the analysis to consider the problem of laminar film condensation on surfaces with a variety of different geometries. An excellent review of these analyses can be found in Merte's study published in 1973 [2]. In addition, a systematic review of the pure forced condensation problem is presented in Fujii's study [3].

The problem of film condensation in a porous medium was originally analyzed by Cheng [4], who used the Darcy model to analyze the condensate flow along both inclined and vertical surfaces. Chen et al. [5] applied a two-zone model to study the problem of moisture migration and condensation within porous insulation layers. Char et al. [6] considered the case of a mixed convection condensate flow along a vertical wall embedded in a porous medium. In practical engineering applications, laminar condensation in a porous medium is not restricted to flat, vertical surfaces, but may occur on surfaces with various geometrical profiles and orientations. As a result, researchers have explored a wide variety of condensation systems, including horizontal disks [7,8], wavy surfaces [9,10], horizontal elliptical tubes [11,12], and so forth. If the effective pore radii of the porous media are very small, these studies have shown that the capillary force induced by the porous medium is instrumental in improving the rate of heat transfer in condensation systems embedded within porous media [13–15]. Moreover, the results have demonstrated that the heat transfer performance of the condensation system can be further improved by applying a suction effect at the wall [16,17].

The current study examines the heat transfer performance of a condensate layer flowing over the external surface of a horizontal elliptical tube embedded in a porous medium with a suction force acting at the tube surface. In modeling the flow of the liquid film, the analysis takes specific account of the variable effect of the surface tension force caused by the non-uniform radius of curvature of the tube surface. By using the separation of variables method and introducing an effective suction function to model the effect of the wall suction on the condensate film thickness, analytical expressions are obtained for the local Nusselt number

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Nomenclature

a	half length of major axis of ellipse
b	half length of minor axis of ellipse
Bo	Bond number defined in Eq. (15)
Cp	specific heat at constant pressure
Da	Darcy number defined in Eq. (18d)
e	ellipticity
f	effective suction function defined in Eq. (24)
g	acceleration of gravity
h	heat transfer coefficient
h_{fg}	heat of vaporization
Ja	Jakob number defined in Eq. (18a)
k	thermal conductivity
K	permeability of porous medium
Nu	Nusselt number defined in Eq. (35a)
Pr_e	effective Prandtl number defined in Eq. (18c)
R	radius of curvature of elliptical tube surface
Ra	Rayleigh number defined in Eq. (18b)
Re_w	Reynolds number defined in Eq. (18e)
s	dimensionless circumference length defined in Eq. (37)
Sw	suction parameter defined in Eq. (18f)
T	temperature
ΔT	saturation temperature minus wall temperature
u	velocity component in x-direction
v	velocity component in y-direction

Greek symbols

δ	condensate film thickness
μ	liquid viscosity
ρ	liquid density
ρ_v	vapor density
σ	surface tension coefficient
α	thermal diffusivity
θ	angle measured from top of tube
ϕ	angle between tangent to tube surface and horizontal direction

Superscripts

—	average quantity
*	dimensionless variable

Subscripts

sat	saturation property
w	quantity at wall
e	effective property

and the condensate film thickness as a function of the Darcy number, the Jakob number, the Rayleigh number, the Bond number and the suction parameter.

2. Analysis

Consider a horizontal, permeable elliptical tube with a wall temperature T_w embedded in a porous medium filled with a pure dry vapor with a uniform temperature T_{sat} (see Fig. 1). As shown, the major axis of the elliptical tube is aligned in the vertical direction and has a length $2a$, while the minor axis is orientated in the horizontal direction and has a length $2b$. Furthermore, the curvilinear coordinates (x, y) are aligned along the tube wall surface and its normal, respectively, and $\phi = \phi(x)$ is the angle between the tangent to the tube wall at position (r,θ) and the horizontal direction. Note that r denotes the radial distance from the centroid of the elliptical tube, while θ is the angle measured from the upper generatrix of the tube. In accordance with first principles, it can be shown that r and θ are related as follows:

$$r = a\left[\frac{(1-e^2)}{(1-e^2\cos^2\theta)}\right]^{1/2}, \tag{1}$$

where e is the ellipticity of the horizontal tube and is defined as

$$e = \frac{\sqrt{\left(a^2-b^2\right)}}{a}. \tag{2}$$

If the wall temperature, T_w, is lower than the saturation temperature, T_{sat}, and the liquid wets the tube surface ideally, a condensate film is formed on the tube surface. The thickness of this liquid film, δ, has a minimum value on the top of the tube, but increases gradually as the liquid flows downward over the tube surface under the effects of gravity and the surface tension force, respectively. As in the classical film condensation problem, the current analysis makes six basic assumptions: (1) the condensate flow is steady and laminar, (2) the inertia force and kinetic energy within the condensation film are negligible and can be ignored, (3) the wall temperature and vapor temperature are uniform and remain constant, (4) the motion of the liquid film in the porous medium is governed by Darcy's law, (5) the physical properties of the dry vapor and the condensate remain constant, and (6) the effective pore radii of the porous medium are sufficiently large that the effects of capillary suction in the porous zone can be neglected.

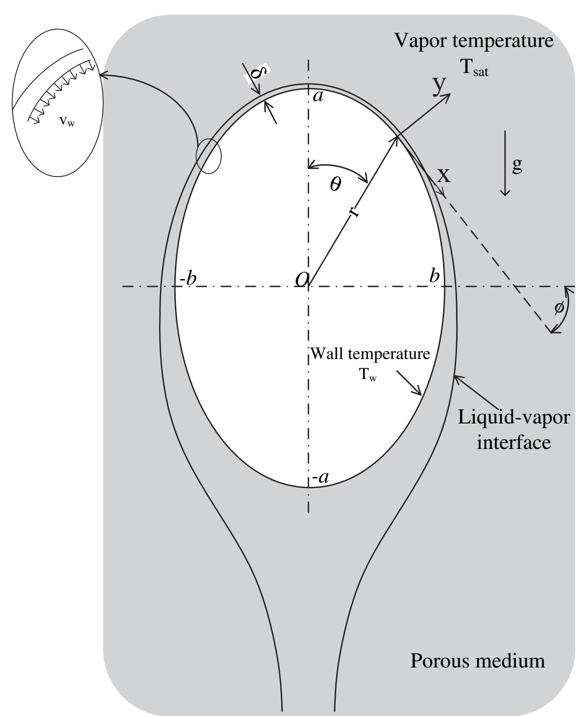

Fig. 1. Schematic diagram of current physical model and corresponding coordinate system.

Note that neglecting the capillary suction effect will induce an error if the effective pore radii are very small. However, in practical problems involving condensation in porous media, the effective pore radii are not very small. Consequently, the effects of capillary suction are generally neglected in most previous studies of heat transfer in a porous medium [4–7,9,11,12,16,18].

Under the assumptions given above, the governing equations for the liquid film subject to boundary layer simplifications are given as follows:

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0. \tag{3}$$

Momentum equation along x-direction:

$$0 = g(\rho-\rho_v)\sin\phi-\frac{\mu_e}{K}u-\frac{\partial P}{\partial x}. \tag{4}$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y} = \alpha_e\frac{\partial^2 T}{\partial y^2}. \tag{5}$$

Note that in Eqs. (3)–(5), K is the intrinsic permeability of the porous medium, u and v are the Darcian velocity components in the x- and y-directions, respectively, α_e is the effective thermal diffusivity of the porous medium saturated with the liquid condensate, and μ_e is the effective dynamic viscosity of the liquid-saturated porous medium. The corresponding boundary conditions are given as follows:

At the tube surface, i.e. $y=0$:

$$v = v_W, \tag{6}$$

$$T = T_W, \tag{7}$$

where v_W is the wall suction velocity.

At the interface of the vapor zone and the liquid film, i.e. $y=\delta$:

$$T = T_{sat}. \tag{8}$$

In free convection problems such as that considered in the current study, the convection terms in Eq. (5) are neglected, and thus the energy equation reduces to

$$0 = \alpha_e\frac{\partial^2 T}{\partial y^2}. \tag{9}$$

In accordance with the boundary conditions specified in Eqs. (7) and (8), the temperature profile within the condensate layer is given by

$$T = T_W+\Delta T\frac{y}{\delta}, \tag{10}$$

where $\Delta T = T_{sat}-T_W$.

Rearranging Eq. (4), the velocity distribution equation can be derived as

$$u = \frac{K}{\mu_e}\left[(\rho-\rho_v)g\sin\phi-\frac{\partial P}{\partial x}\right]. \tag{11}$$

In the current problem, the density of the vapor is relatively small compared to that of the condensate, and thus the term $(\rho-\rho_v)$ can be approximated as ρ. Moreover, the thickness of the condensate layer is significantly smaller than the radius of curvature of the tube surface, and hence the pressure gradient caused by the surface tension effect can be approximated as

$$-\frac{\partial P}{\partial x} = \frac{\sigma}{R^2}\frac{\partial R}{\partial x}, \tag{12}$$

where R is the radius of curvature of the elliptical tube at position (r,θ) and can be derived as

$$R = \frac{a}{\sqrt{(1-e^2)}}\left[\frac{1+e^2(e^2-2)\cos^2\theta}{1-e^2\cos^2\theta}\right]^{3/2}. \tag{13}$$

Substituting Eqs. (12) and (13) into Eq. (11), yields

$$u = \frac{\rho g K}{\mu_e}[\sin\phi+Bo(x)], \tag{14}$$

where

$$Bo(x) = \frac{-1}{\rho g}\frac{\partial P}{\partial x} = \frac{1}{Bo}\left(\frac{a}{R}\right)^2\frac{\partial R}{\partial x}, \tag{15}$$

in which Bo is the Bond number and is given by $Bo=\rho g a^2/\sigma$.

Applying Nusselt's classical analysis method, the overall energy balance in the liquid film can be written as

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{\int_0^\delta \rho u\left(h_{fg}+Cp(T_{sat}-T)\right)\mathrm{d}y\right\}\mathrm{d}x + \rho\left(h_{fg}+Cp\Delta T\right)v_W\mathrm{d}x = k_e\frac{\partial T}{\partial y}\mathrm{d}x, \tag{16}$$

where k_e is the effective thermal conductivity of the porous medium when saturated with condensate.

The right-hand side of Eq. (16) represents the energy transferred from the liquid film to the tube surface. Meanwhile, the first term on the left-hand side of the equation describes the net energy flux across the liquid film (from x to $x+dx$) while the second term represents the net energy sucked out of the system by the permeable tube.

Substituting Eqs. (10) and (14) into Eq. (16) yields

$$\frac{\rho^2 gK\left(h_{fg}+\frac{1}{2}Cp\Delta T\right)}{\mu_e}\frac{\mathrm{d}}{\mathrm{d}x}\{\delta(\sin\phi+Bo(x))\} + \rho\left(h_{fg}+Cp\Delta T\right)v_W = k_e\frac{\Delta T}{\delta}. \tag{17}$$

For analytical convenience, the following dimensionless parameters are defined:

$$Ja = \frac{Cp\Delta T}{h_{fg}+\frac{1}{2}Cp\Delta T}, \tag{18a}$$

$$Ra = \frac{\rho^2 g Pr_e a^3}{\mu_e^2}, \tag{18b}$$

$$Pr_e = \frac{\mu_e Cp}{k_e} \tag{18c}$$

$$Da = \frac{K}{a^2}, \tag{18d}$$

$$Re_W = \frac{\rho v_W a}{\mu_e}, \tag{18e}$$

$$Sw = \left(1+\frac{1}{2}Ja\right)Re_W\frac{Pr_e}{RaDa}, \tag{18f}$$

where Ja is the Jakob number; Pr_e is the effective Prandtl number; Ra is the effective Rayleigh number; Da is the Darcy number; Re_w is the Reynolds number; and Sw is the suction parameter.

Substituting Eqs. 18(a)–18(f) into Eq. (17) yields

$$\delta\frac{d}{dx}\{\delta(\sin\phi+Bo(x))\}+Sw\delta = \frac{Ja}{Ra}\frac{a}{Da}. \tag{19}$$

According to Yang [18], parameters x, θ and ϕ of an elliptical tube are related as follows:

$$\tan\theta = (1-e^2)\tan\phi, \tag{20}$$

$$dx = \frac{a(1-e^2)}{\left(1-e^2\sin^2\phi\right)^{3/2}}d\phi. \tag{21}$$

Substituting Eqs (1), (13) and (20) into Eq. (15) yields

$$Bo(x) = Bo(\phi) = \frac{1}{Bo}\times\frac{3e^2}{2}\left(\frac{1-e^2\sin^2\phi}{1-e^2}\right)^2\times\sin(2\phi). \tag{22}$$

Substituting Eqs. (21) and (22) into Eq. (19) and introducing the dimensionless liquid film thickness, $\delta^* = \delta/a$, Eq. (19) can be rewritten as

$$\delta^*\frac{\left(1-e^2\sin^2\phi\right)^{3/2}}{(1-e^2)}\frac{d}{d\phi}\left\{\delta^*(\sin\phi+Bo(\phi))\right\}+Sw\delta^* = \frac{Ja}{RaDa}. \tag{23}$$

In other words, the dimensionless liquid film thickness, $\delta^*(\phi)$, is a function of the ellipticity, e, the Jakob number, Ja, the Rayleigh number, Ra, the Darcy number, Da, the Bond number, Bo, and the suction parameter, Sw. However, it is difficult to solve δ^* directly. Accordingly, the present study introduces an effective suction function, f, to model the effect of the wall suction on the thickness of the condensate film, i.e.

$$\frac{\delta^*}{\delta^*|_{Sw=0}} = 1-f, \tag{24}$$

or

$$\delta^* = (1-f)\times\delta^*|_{Sw=0}, \tag{25}$$

where $\delta^*|_{Sw=0}$ is the dimensionless liquid film thickness at $Sw = 0$.

Although the dimensionless liquid film thickness reduces as a result of the wall suction effect ($\delta^* \le \delta^*|_{Sw=0}$ or $f \ge 0$), it cannot physically attain a negative value ($\delta^* \ge 0$ or $f \le 1$). In other words, f falls within the range $0 \le f \le 1$, where $f = 0$ at $Sw = 0$.

In the absence of a wall suction effect, i.e. $Sw = 0$, Eq. (23) can be rewritten as

$$\delta^*|_{Sw=0}\frac{d}{d\phi}\left\{\delta^*|_{Sw=0}(\sin\phi+Bo(\phi))\right\} = \frac{(1-e^2)}{\left(1-e^2\sin^2\phi\right)^{3/2}}\times\frac{Ja}{RaDa}. \tag{26}$$

Applying the separation of variables method, Eq. (26) can be reformulated as

$$\left\{\delta^*|_{Sw=0}(\sin\phi+Bo(\phi))\right\}d\left\{\delta^*|_{Sw=0}(\sin\phi+Bo(\phi))\right\} = \frac{(1-e^2)}{\left(1-e^2\sin^2\phi\right)^{3/2}}\times\frac{Ja}{RaDa}\times(\sin\phi+Bo(\phi))d\phi. \tag{27}$$

Integrating both sides of Eq. (27), $\delta^*|_{Sw=0}$ can be derived as

$$\frac{1}{2}\left\{\delta^*|_{Sw=0}(\sin\phi+Bo(\phi))\right\}^2 = \frac{Ja}{RaDa}(1-e^2)\times\int_o^\phi\frac{\sin\phi+Bo(\phi)}{\left(1-e^2\sin^2\phi\right)^{3/2}}d\phi \tag{28}$$

or

$$\delta^*|_{Sw=0} = \delta^*(\phi)|_{Sw=0} = \frac{1}{(\sin\phi+Bo(\phi))}\times\left\{\frac{2Ja}{RaDa}\left(1-e^2\right)\times\int_o^\phi\frac{\sin\phi+Bo(\phi)}{\left(1-e^2\sin^2\phi\right)^{3/2}}d\phi\right\}^{1/2}. \tag{29}$$

Equation (29) provides the analytical solution for the dimensionless film thickness in the absence of a wall suction effect. If the values of e, Ja, Pr_e, Ra, Da and Bo are all given, the value of $\delta^*|_{Sw=0}$ for any given ϕ can be derived using a simple numerical integration method.

Substituting Eq. (29) into Eq. (25), the dimensionless liquid film thickness δ^* can be rewritten as

$$\delta^* = (1-f)\times\delta^*|_{Sw=0} = \frac{(1-f)}{(\sin\phi+Bo(\phi))}\times\left\{\frac{2Ja}{RaDa}\left(1-e^2\right)\times\int_o^\phi\frac{\sin\phi+Bo(\phi)}{\left(1-e^2\sin^2\phi\right)^{3/2}}d\phi\right\}^{1/2}. \tag{30}$$

Substituting Eq. (25) into Eq. (23), Eq. (23) can be rewritten in the form of the following first-order differential equation:

$$\left(\delta^*|_{Sw=0}\right)^2(\sin\phi+Bo(\phi))(f-1)\frac{df}{d\phi}+\left(\delta^*|_{Sw=0}\right)\left(f^2-2f\right)\times\frac{d}{d\phi}\left\{\left(\delta^*|_{Sw=0}\right)(\sin\phi+Bo(\phi))\right\}+\frac{(1-e^2)}{\left(1-e^2\sin^2\phi\right)^{3/2}}\times Sw\times(1-f)\times\delta^*|_{Sw=0} = 0. \tag{31}$$

The corresponding boundary condition is given by the following symmetrical function.

$$df/d\phi = 0 \quad \text{at}\ \phi = 0 \tag{32}$$

If the values of e, Ja, Ra, Da, Bo and Sw are given, the numerical solutions for the effective suction function, $f(\phi)$, can be obtained using a shooting method. The solution procedure commences by substituting the initial boundary condition at the top of the tube given in Eq. (32) into Eq. (31), yielding

$$Af^2(0)+Bf(0)+C = 0 \tag{33a}$$

or

$$f(0) = \frac{-B\pm\sqrt{B^2-4AC}}{2A}, \tag{33b}$$

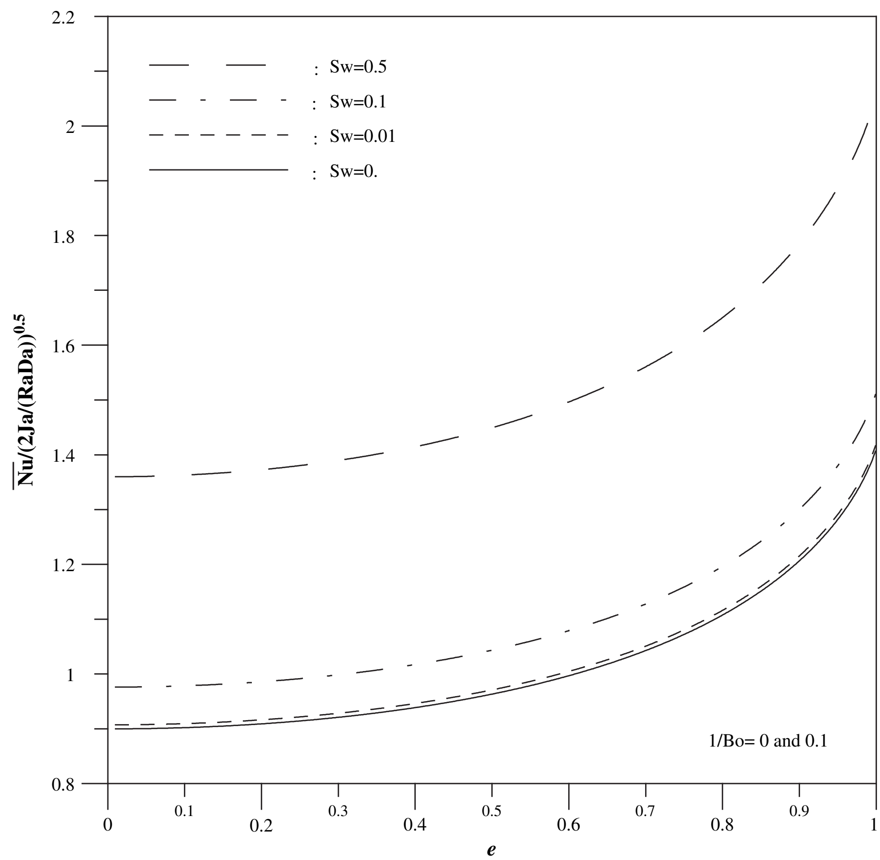

Fig. 2. Variation of $\overline{Nu}$ with e as function of Sw and Bo.

where

$$A = \delta^*|_{Sw=0} \times \frac{d}{d\phi}\left\{\left(\delta^*|_{Sw=0}\right)(\sin\phi + Bo(\phi))\right\}, \tag{34a}$$

$$B = -2\delta^*|_{Sw=0} \times \frac{d}{d\phi}\left\{\left(\delta^*|_{Sw=0}\right)(\sin\phi + Bo(\phi))\right\} - \left(1-e^2\right) \times Sw \times \delta^*|_{Sw=0}, \tag{34b}$$

$$C = \left(1-e^2\right) \times Sw \times \delta^*|_{Sw=0}. \tag{34c}$$

As stated above, the effective suction parameter f falls within the range $0 \le f \le 1$. For the particular case of $e \to 1$, the positive sign of the $\pm$ sign in Eq. (33b) results in $f(0) > 1$, which contravenes the range assigned to f. In other words, $f(0)$ can only be solved by using the negative sign in Eq. (33b). Given the values of $\delta^*|_{Sw=0}$ and $Bo(\phi)$, the variation of $f(\phi)$ can then be calculated at all points of interest along the circumference of the tube.

In condensation heat transfer problems, the Nusselt number is one of the most important considerations. The local Nusselt number is defined as

$$Nu = \frac{ha}{k_e}, \tag{35a}$$

where

$$h = \frac{k_e}{\delta} = \frac{k_e}{(1-f) \times \delta^*|_{Sw=0} \times a}. \tag{35b}$$

Meanwhile, the mean Nusselt number is defined as

$$\overline{Nu} = \frac{1}{\pi}\int_0^{\pi} Nu\,d\phi. \tag{36}$$

In presenting the distributions of the dimensionless liquid film thickness and the local Nusselt number along the tube surface, it is inappropriate to use ϕ as the position coordinate since ϕ indicates the angle between the horizontal direction and the tangent to the tube wall at position (r,θ) rather than the actual position on the tube surface. Hence, the present study defines a dimensionless parameter s to represent the corresponding dimensionless circumference length from the top of the tube. From Eq. (21), it can be shown that s is given by

Table 1
Comparison of solutions for $\overline{Nu}$ obtained in present study with those presented by Cheng [4] and Chiou et al. [11].

	$\overline{Nu}$ (previous studies)	$\overline{Nu}$ (present study at $Sw = 0$)
Vertical plate ($e \to 1$)	$\overline{Nu} = \sqrt{2} \times (RaDa/2Ja)^{1/2}$, (Cheng)	$\overline{Nu} = 1.4 \times (RaDa/2Ja)^{1/2}$
Circular tube ($e = 0$)	$\overline{Nu} = (2\sqrt{2}/\pi) \times (RaDa/2Ja)^{1/2}$, (Chiou et al.)	$\overline{Nu} = 0.9 \times (RaDa/2Ja)^{1/2}$

$$s = \frac{x(\phi)}{x(\pi)} = \frac{\int_0^{\phi} \left(1 - e^2 \sin^2 \phi\right)^{-3/2} \mathrm{d}\phi}{\int_0^{\pi} \left(1 - e^2 \sin^2 \phi\right)^{-3/2} \mathrm{d}\phi}. \tag{37}$$

3. Results & discussions

Fig. 2 shows the effect of the tube ellipticity, e, on the mean Nusselt number, $\overline{Nu}$, for different values of the suction parameter, Sw. As expected, the results show that the mean Nusselt number increases as the suction effect is increased. Cheng [4] employed a two-zone model to investigate the problem of film condensation on an inclined surface in a porous medium in the absence of wall suction effects (i.e. $Sw = 0$ in the present study). Meanwhile, Chiou et al. [11] used a novel transformation method to investigate the problem of film condensation on a horizontal elliptical tube in a porous medium with no suction effect at the wall. Table 1 compares the solutions for $\overline{Nu}$ presented by Cheng [4] and Chiou et al. [11] for a vertical plate (i.e. $e \to 1$) and a circular tube (i.e. $e = 0$), respectively, with those derived in the current study for $Sw = 0$. Note that the parameters defined in [4] and [11] differ from those used in the current analysis. Therefore, to enable a direct comparison with the current results, the formulations presented in Table 1 for Chiou et al. and Cheng have been transformed from their original formats and expressed in terms of the current parameters. It is observed that a good agreement exists between the current results and those presented in the literature.

Fig. 2 also shows that the mean Nusselt number increases as the tube ellipticity, e, increases. Finally, Fig. 2 shows that even though two different values of the surface tension parameter are considered (i.e. $1/Bo = 0$ and 0.1, respectively), the figure contains just four lines (corresponding to $Sw = 0$, 0.01, 0.1 and 0.5, respectively). In other words, for a given value of Sw, the corresponding profile represents the superposition of the results obtained for both values of $1/Bo$. Thus, the results suggest that the mean Nusselt number is independent of the surface tension parameter Bo.

The effect of the tube's ellipticity on the local film thickness and the corresponding local Nusselt number, is illustrated in Figs. 3–5 for $e = 0$, 0.7 and 0.9, respectively. In the case of a circular tube, the surface has a constant radius of curvature. Therefore, from Eq. (15), it is apparent that $Bo(x) = Bo(\phi) = 0$. In other words, the surface tension force has no effect on either the dimensionless liquid film thickness or the heat transfer performance of the circular tube. Consequently, it can be inferred that the dimensionless liquid film thickness, δ^*, and local Nusselt number, Nu, are functions only of Ja, Da, Ra, s, and Sw. Fig. 3 shows the distributions of the dimensionless film thickness and the local Nusselt number along the surface of a circular tube for various values of the suction parameter, Sw. In general, the results show that the dimensionless liquid film thickness decreases, while the local Nusselt number increases, with increasing Sw. Furthermore, it can be seen that the dimensionless liquid film thickness has a minimum value at the top of the tube (i.e. $s = 0$) and increases with increasing s. This result is to be expected since the current study considers the problem of falling film condensation over the external surface of a horizontal tube. In such problems, the gravity effect causes the film to have a minimum thickness (and thus a maximum Nusselt number) in its uppermost position (i.e. the upper surface of the tube). The thickness of the film increases gradually as it flows downward over the tube surface and tends toward an infinite value at the lower surface of the tube, i.e. $s = 1$, corresponding to a Nusselt number of zero.

For an elliptical tube, the radius of curvature of the surface is non-uniform, and therefore the surface tension force exerts a variable effect on the heat transfer performance depending on the particular position of the tube surface. Fig. 4a shows the variation of

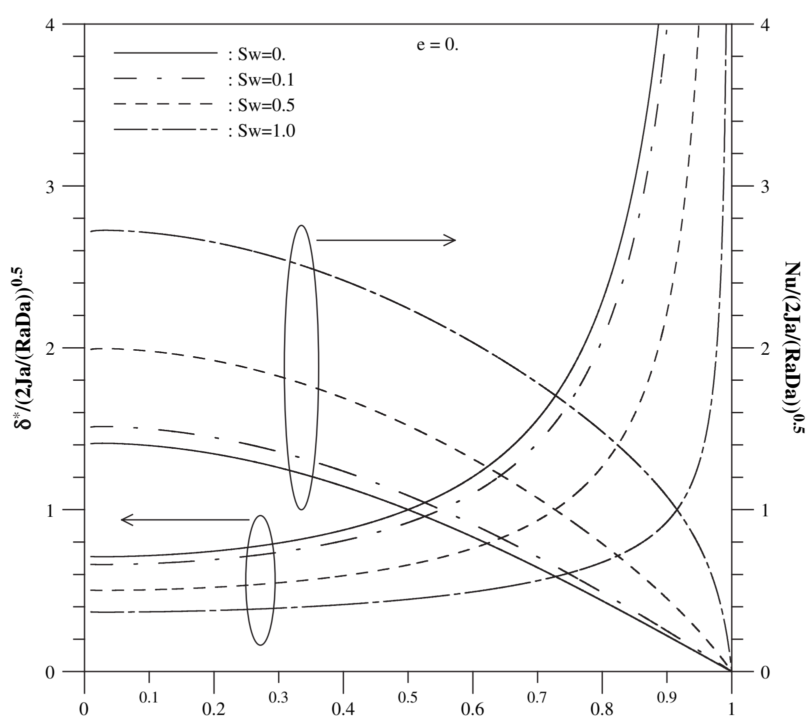

Fig. 3. Variations of dimensionless film thickness and Nu along surface of circular tube as function of Sw.

the dimensionless film thickness along the surface of a tube with an ellipticity of $e = 0.7$ for various values of $1/Bo$ and Sw. As in Fig. 3, it can be seen that the dimensionless liquid film thickness decreases with increasing Sw. In addition, for a given value of Sw, the liquid film thickness on the upper surface of the tube is thinner for the case of $1/Bo = 0.1$ than for the case where the surface tension effect is neglected, i.e. $1/Bo = 0$. Conversely, on the lower surface of the tube, the liquid film has a greater thickness for the case of $1/Bo = 0.1$ than for $1/Bo = 0$. The reason for this apparently conflicting tendency is that on the upper surface of the tube, the surface tension force has a positive value since the radius of curvature increases with increasing x (see Fig. 1), i.e. $-\partial P/\partial x = (\sigma/R^2)\,(\partial R/\partial x) > 0$, and as a result, the liquid film is pulled down over the side surfaces of the tube, causing the thickness of the film on the upper surface to reduce. Conversely, on the lower surface of the tube, the surface tension force has a negative value, i.e. $-\partial P/\partial x = (\sigma/R^2)\,(\partial R/\partial x) < 0$, and thus the liquid film is effectively pulled in the upward direction, causing an accumulation of the condensate liquid and a corresponding increase in the thickness of the liquid film. Fig. 4(b) shows the variation of the local Nusselt number over the surface of the tube considered in Fig. 4(a). It can be seen that the Nusselt number increases with an increasing suction parameter Sw. In addition, the surface tension force effect causes the local Nusselt number to increase on the upper surface of the tube, but to decrease on the lower surface of the tube. From a mean Nusselt number perspective, the positive and negative effects of the surface

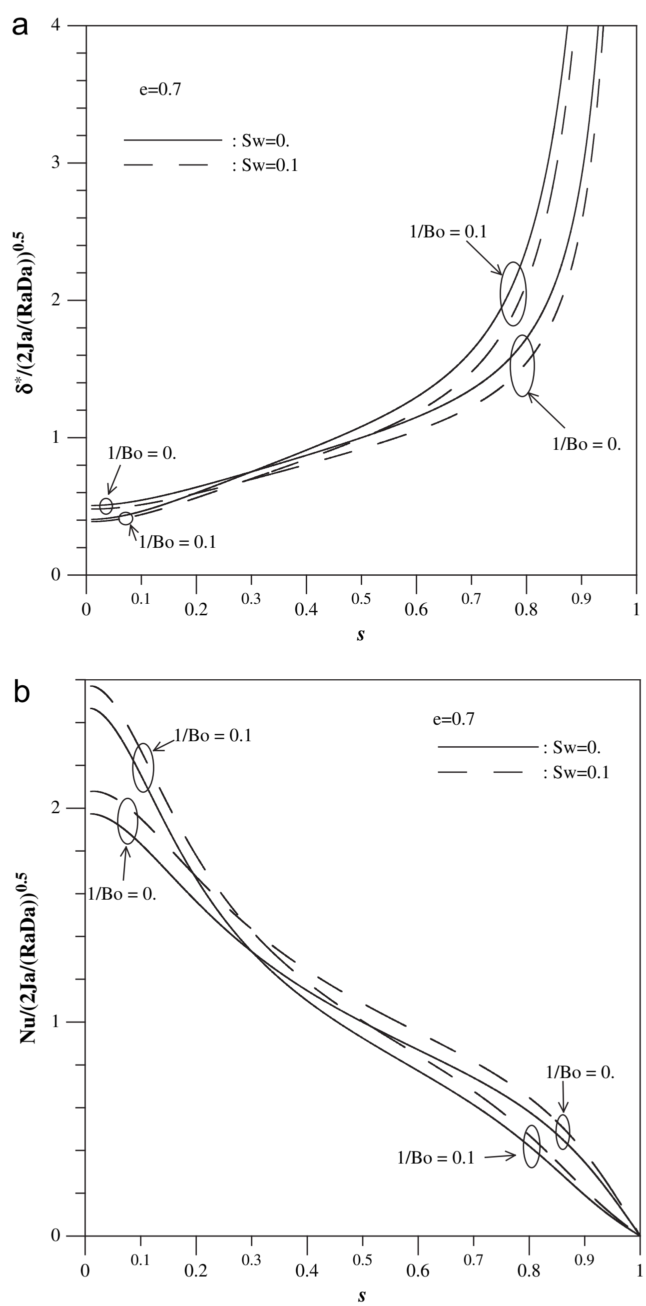

Fig. 4. a. Variation of dimensionless film thickness along tube surface as function of Sw and Bo for $e = 0.7$. b. Variation of Nu along tube surface as function of Sw and Bo for $e = 0.7$.

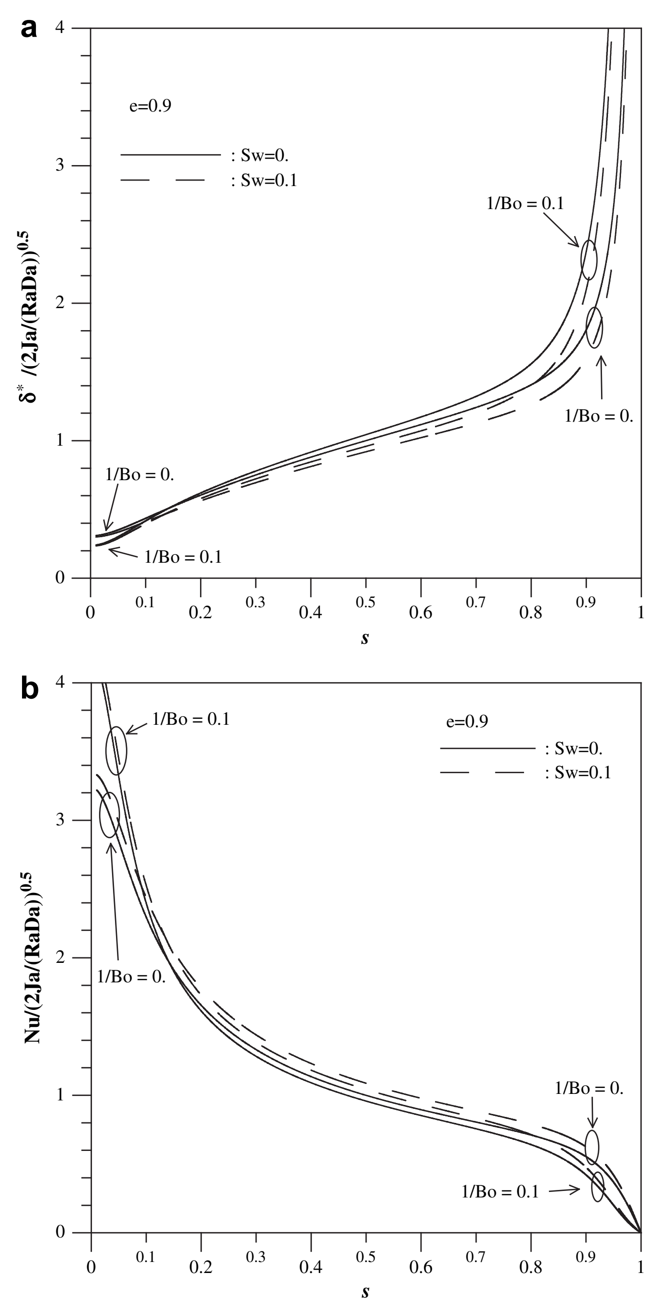

Fig. 5. a. Variation of dimensionless film thickness along tube surface as function of Sw and Bo for $e = 0.9$. b. Variation of Nu along tube surface as function of Sw and Bo for $e = 0.9$.

tension force cancel one another out, and therefore the surface tension parameter has a negligible effect on the mean Nusselt number (as shown in Fig. 2).

Fig. 5a and b shows the distributions of the dimensionless film thickness and the Nusselt number, respectively, along the external surface of a tube with an ellipticity of $e = 0.9$. It is apparent that the general tendencies of the two figures are similar to those shown in Fig. 4a and b for a tube with an ellipticity of $e = 0.7$. In other words, on the upper surface of the tube, the liquid film thickness decreases and the Nusselt number increases as $1/Bo$ increases due to the positive effect of the surface tension force, whereas on the lower surface of the tube, the liquid film thickness increases and the Nusselt number decreases as $1/Bo$ increases due to the negative effect of the surface tension force.

Comparing Figs. 3, 4a and 5a, it is observed that the dimensionless film thickness decreases as the ellipticity, e, of the tube increases. This finding is reasonable since as the ellipticity of the horizontal tube increases, its profile converges toward that of a vertical plate. Under this geometric condition, the effect of the gravitational force acting on the condensate layer increases, and thus the thickness of the liquid film reduces.

4. Conclusion

This study has analyzed the problem of a two-dimensional laminar film condensation flow over the external surface of a horizontal elliptical tube embedded in a porous medium with suction at the tube surface. In performing the analysis, Darcy's law has been employed to model the flow of the liquid film within the porous medium under the combined effects of the gravitational force and the surface tension force. In addition, an effective suction function, f, has been introduced to model the effect of the suction force at the wall on the thickness of the condensate film, thereby enabling both the condensate film thickness and the local Nusselt number to be derived as functions of the Darcy number, the Jakob number, the Rayleigh number, the Bond number and the suction parameter. In general, the results presented in this study have shown that the heat transfer performance can be enhanced by applying a suction effect at the wall. Furthermore, it has been shown that although the surface tension force has a negligible effect on the mean Nusselt number, it exerts a significant effect on the local Nusselt number and must therefore be taken into consideration.

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